

## Question

A spectroscopic instrument can resolve two nearby wavelengths  $\lambda$  and  $\lambda + \Delta\lambda$  if  $\lambda/\Delta\lambda$  is smaller than 8000. This is used to study the spectral lines of visible series of hydrogen (protium). Approximately how many lines will be resolved by the instrument.

## Solution

### VISIBLE SERIES :BALMER SERIES

$\lambda$  = wavelength

$d\lambda$  = change in wavelength

$$1/\lambda = R\{1/m^2 - 1/n^2\} \text{ -----(1)}$$

differentiating

$$d\lambda / \lambda^2 = -2R/n^3 dn$$

negative sign shows decrease in wavelength with increase of  $n$ , taking modulus

$$d\lambda / \lambda^2 = 2R/n^3 dn$$

from (1)

$$(d\lambda / \lambda) R\{1/m^2 - 1/n^2\} = 2R/n^3 dn \text{ -----(2)}$$

given  $\lambda / d\lambda < 8000$

$$\text{or, } d\lambda / \lambda > 1/8000$$

From (2)

$$d\lambda / \lambda = dn \{2m^2 / (n^2 - m^2)n\} > 1/8000$$

$dn=1$  because two nearby wavelength can have  $\min(dn)=1$

$$m^2 / (n^2 - m^2)n > 1/16000$$

on simplification

$$(n^3 - nm^2) / m^2 < 16000$$

for Balmer series,  $m=2$

$$(n^3 - 4n) / 4 < 16000$$

$$\text{or, } n^3 - 4n < 64000$$

$$\text{or, } n^3 - 40^3 < 4n$$

which is satisfied till  $n=40$

therefore, total number of lines =  $40-2=38$

ans: total number of lines = 38