L = $\displaystyle \operatorname*{Lt}_{\substack{x\to a\\ y\to a\\ z\to a}} \frac{x^p\left(y^q - z^q\right) + y^p\left(z^q - x^q\right) + z^p\left(x^q - y^q\right)}{x^r\left(y^s - z^s\right) + y^r\left(z^s - x^s\right) + z^r\left(x^s - y^s\right)}$

= $\displaystyle \operatorname*{Lt}_{\substack{x\to a\\ y\to a\\ z\to a}} \frac{\Sigma x^p\left(y^q - z^q\right)}{\Sigma x^r\left(y^s - z^s\right)}$

where Σ denotes not the normal summation but the cyclic symmetric function. Eg.
Σa(b-c) = a(b-c) + b(c-a) + c(a-b)
Now let x = h_1 + a ; y = h_2 + a ; z = h_3 + a.
So

L = $\displaystyle \operatorname*{Lt}_{\substack{h_1\to 0\\ h_2\to 0\\ h_3\to 0}} \frac{\Sigma(h_1+a)^p\left\{(h_2+a)^q - (h_3+a)^q\right\}}{\Sigma(h_1+a)^r\left\{(h_2+a)^s - (h_3+a)^s\right\}}$

= $\displaystyle \operatorname*{Lt}_{\substack{h_1\to 0\\ h_2\to 0\\ h_3\to 0}} \frac{a^{p+q}\Sigma\left[\left(1+\frac{h_1}{a}\right)^p\left\{\left(1+\frac{h_2}{a}\right)^q - \left(1+\frac{h_3}{a}\right)^q\right\}\right]}{a^{r+s}\Sigma\left[\left(1+\frac{h_1}{a}\right)^r\left\{\left(1+\frac{h_2}{a}\right)^s - \left(1+\frac{h_3}{a}\right)^s\right\}\right]}$

= $\displaystyle \operatorname*{Lt}_{\substack{h_1\to 0\\ h_2\to 0\\ h_3\to 0}} \frac{a^{p+q-(r+s)}N}{D}$

where N = $\displaystyle \Sigma\left[\left(1+\frac{h_1}{a}\right)^p\left\{\left(1+\frac{h_2}{a}\right)^q - \left(1+\frac{h_3}{a}\right)^q\right\}\right]$

D = $\displaystyle \Sigma\left[\left(1+\frac{h_1}{a}\right)^r\left\{\left(1+\frac{h_2}{a}\right)^s - \left(1+\frac{h_3}{a}\right)^s\right\}\right]$

Now

$$N = \Sigma\left[\left(1+\frac{h_1}{a}\right)^p\left\{\left(1+\frac{h_2}{a}\right)^q-\left(1+\frac{h_3}{a}\right)^q\right\}\right]$$

$= \Sigma(AB)$

where $A = \left(1+\frac{h_1}{a}\right)^p$ and $B = \left\{\left(1+\frac{h_2}{a}\right)^q-\left(1+\frac{h_3}{a}\right)^q\right\}$

Now applying binomial expansion on A

$$A = \left(1+\frac{ph_1}{a}+\frac{p(p-1)h_1^{\ 2}}{2a^2}+\frac{p(p-1)(p-2)h_1^{\ 3}}{6a^3}+\frac{p(p-1)(p-2)(p-3)h_1^{\ 4}}{24a^4}\ldots\ldots\ldots\right)$$

Applying binomial expansion on B

$$B = \left\{\frac{q(h_2-h_3)}{a}+\frac{q(q-1)(h_2^{\ 2}-h_3^{\ 2})}{2a^2}+\frac{q(q-1)(q-2)(h_2^{\ 3}-h_3^{\ 3})}{6a^3}+\frac{q(q-1)(q-2)(q-3)(h_2^{\ 4}-h_3^{\ 4})}{24a^4}\ldots\ldots\right\}$$

So

$N = \Sigma(AB)$

$$= \left(\frac{q}{a}\Sigma(h_2-h_3)+\frac{q(q-1)}{2a^2}\Sigma(h_2^{\ 2}-h_3^{\ 2})+\frac{q(q-1)(q-2)}{6a^3}\Sigma(h_2^{\ 3}-h_3^{\ 3})+\ldots\ldots\right)$$

$$+\left(\frac{pq}{a^2}\Sigma h_1(h_2-h_3)+\frac{pq(q-1)}{2a^3}\Sigma h_1(h_2^{\ 2}-h_3^{\ 2})+\frac{pq(q-1)(q-2)}{6a^4}\Sigma h_1(h_2^{\ 3}-h_3^{\ 3})+\ldots\ldots\right)$$

$$+\left(\frac{pq(p-1)}{2a^3}\Sigma h_1^{\ 2}(h_2-h_3)+\frac{pq(p-1)(q-1)}{4a^4}\Sigma h_1^{\ 2}(h_2^{\ 2}-h_3^{\ 2})+\frac{pq(p-1)(q-1)(q-2)}{12a^5}\Sigma h_1^{\ 2}(h_2^{\ 3}-h_3^{\ 3})\ldots\ldots\right)$$

$$+\left(\frac{pq(p-1)(p-2)}{6a^4}\Sigma h_1^{\ 3}(h_2-h_3)+\frac{pq(p-1)(p-2)(q-1)}{12a^5}\Sigma h_1^{\ 3}(h_2^{\ 2}-h_3^{\ 2})+\frac{pq(p-1)(p-2)(q-1)(q-2)}{36a^6}\Sigma h_1^{\ 3}(h_2^{\ 3}-h_3^{\ 3})+\ldots\right)$$

+ Σ terms containing degrees more than 3 in h_1, h_2, h_3

Now, by expanding the following it can be found that

$\Sigma h_1(h_2-h_3) = \Sigma h_1^2(h_2^2-h_3^2) = \Sigma h_1^3(h_2^3-h_3^3) = 0$ i.e. $\Sigma h_1^n(h_2^n-h_3^n) = 0$.

$\Sigma h_1(h_2^2-h_3^2) = -\Sigma h_1^2(h_2-h_3)$; $\Sigma h_1(h_2^3-h_3^3) = -\Sigma h_1^3(h_2-h_3)$; i.e. $\Sigma h_1^m(h_2^n-h_3^n) = -\Sigma h_1^n(h_2^m-h_3^m)$

By taking all these in consideration we can write

$$N = (0 + 0 + 0 \ldots\ldots)$$

$$+\left(0+\frac{pq(q-1)}{2a^3}\Sigma h_1\left(h_2^{\,2}-h_3^{\,2}\right)+\Sigma terms\ \ containing\ \ degrees\ \ more\ \ than\ \ 3\ \ in\ \ h_1,h_2,h_3\right)$$

$$+\left(\frac{pq(q-1)}{2a^3}\left\{-\Sigma h_1\left(h_2^{\,2}-h_3^{\,2}\right)\right\}+\Sigma terms\ \ containing\ \ degrees\ \ more\ \ than\ \ 3\ \ in\ \ h_1,h_2,h_3\right)$$

+ Σ terms containing degrees more than 3 in h_1,h_2,h_3 .

$$= \left(\frac{pq(q-p)}{2a^3}\left\{\Sigma h_1\left(h_2^{\,2}-h_3^{\,2}\right)\right\}\right) + \Sigma \text{ terms containing degrees more than 3 in } h_1,h_2,h_3$$

$$= \left(\frac{pq(q-p)}{2a^3}\left\{\Sigma h_1\left(h_2^{\,2}-h_3^{\,2}\right)\right\}\right) + F(h_1,h_2,h_3)$$

where $F(h_1,h_2,h_3)$ is a cyclic symmetrical expression in h_1,h_2,h_3 with degree more than 3.

Since $F(h_1,h_2,h_3)$ is a cyclic symmetrical expression in h_1,h_2,h_3 hence if we put either $h_1=h_2$ or $h_2=h_3$ or $h_3=h_1$ then we will surely get $F(h_1,h_2,h_3) = 0$ i.e. $(h_1- h_2)(h_2- h_3)(h_3- h_1)$ is a factor of $F(h_1,h_2,h_3)$.

So we can write $F(h_1,h_2,h_3) = (h_1- h_2)(h_2- h_3)(h_3- h_1)\, G(h_1,h_2,h_3)$

where $G(h_1,h_2,h_3)$ is a polynomial in h_1,h_2,h_3. Also $G(h_1,h_2,h_3)$ does not have any constant term since $F(h_1,h_2,h_3)$ has a degree more than 3. So the least degree term of $G(h_1,h_2,h_3)$ is of degree 1 i.e. either h_1 or h_2 or h_3.

So when $h_1=h_2=h_3=0$ then $G(h_1,h_2,h_3) = 0$ i.e. $\underset{\substack{h_1\to 0\\ h_2\to 0\\ h_3\to 0}}{Lt} G(h_1,h_2,h_3) = 0$.

Also on expanding it is found that

$\Sigma h_1(h_2^2-h_3^2) = (h_1- h_2)(h_2- h_3)(h_3- h_1)$

So

$$N = \left(\frac{pq(q-p)}{2a^3} + G(h_1,h_2,h_3)\right)(h_1- h_2)(h_2- h_3)(h_3- h_1)$$

Similarly

$$D = \left(\frac{rs(s-r)}{2a^3} + H(h_1,h_2,h_3)\right)(h_1- h_2)(h_2- h_3)(h_3- h_1)$$

where $H(h_1,h_2,h_3)$ is a polynomial in h_1,h_2,h_3.

So when $h_1=h_2=h_3=0$ then $H(h_1,h_2,h_3) = 0$ i.e. $\underset{\substack{h_1\to 0\\ h_2\to 0\\ h_3\to 0}}{Lt} H(h_1,h_2,h_3) = 0$.

Now substituting the values of N & D in L we get

$$L = \underset{\substack{h_1 \to 0 \\ h_2 \to 0 \\ h_3 \to 0}}{Lt} \frac{a^{p+q-(r+s)}\left(\frac{pq(q-p)}{2a^3} + G(h_1, h_2, h_3)\right)}{\left(\frac{rs(s-r)}{2a^3} + H(h_1, h_2, h_3)\right)}$$

$$= a^{(p+q)-(r+s)} \frac{\underset{\substack{h_1 \to 0 \\ h_2 \to 0 \\ h_3 \to 0}}{Lt} \frac{pq(q-p)}{2a^3} + \underset{\substack{h_1 \to 0 \\ h_2 \to 0 \\ h_3 \to 0}}{Lt} G(h_1, h_2, h_3)}{\underset{\substack{h_1 \to 0 \\ h_2 \to 0 \\ h_3 \to 0}}{Lt} \frac{rs(s-r)}{2a^3} + \underset{\substack{h_1 \to 0 \\ h_2 \to 0 \\ h_3 \to 0}}{Lt} H(h_1, h_2, h_3)}$$

$$= a^{(p+q)-(r+s)} \frac{pq(q-p)}{rs(s-r)} \quad = a^{(p+q)-(r+s)} \frac{pq(p-q)}{rs(r-s)}$$