Answer to crack problem 51

$$P \xrightarrow{\lambda_p} D \xrightarrow{\lambda_d} X$$

$-d[P]/dt = \lambda_p[P]$ = rate of formation of D

rate of decrease of D= $-d[D]/dt = \lambda_d[D]$

rate of change (formation) of D=$\lambda_p[P]- \lambda_d[D]$

$P_t=P_oe^{-\lambda pt}$

$$d[D]/dt = \lambda_p P_oe^{-\lambda pt} - \lambda_d[D]$$

$$d[D]/dt + \lambda_d[D] = \lambda_p P_oe^{-\lambda pt}$$

integrating with conditions (at t=0, [D]=0)

I.F=$e^{\lambda dt}$

$$[D] e^{\lambda dt} = \int \lambda_p P_oe^{-\lambda pt+\lambda d\, t}dt$$

$$[D] e^{\lambda dt} = \frac{\lambda_p Po(e^{-\lambda pt+\lambda d\, t}-1)}{\lambda_p-\lambda_d}$$

$$[D] = \frac{\lambda_p Po(e^{-\lambda pt} - e^{-\lambda dt})}{\lambda_p-\lambda_d}$$

for max. [D] $d[D]/dt = 0$ $d^2[D]/dt^2 < 0$

from above we get, differentiating

$$d[D]/dt = \frac{\lambda_p Po\{(-e^{-\lambda_p t}\lambda_p) - (-e^{-\lambda_d t}\lambda_d)\}}{\lambda_p - \lambda_d} = 0$$

$$\Rightarrow \{(-e^{-\lambda_p t}\lambda_p) - (-e^{-\lambda_d t}\lambda_d)\} = 0$$

$$e^{-\lambda_d t}\lambda_d - e^{-\lambda_p t}\lambda_p = 0$$

taking log, we get

$$t_{max} = \frac{\ln\left(\frac{\lambda_D}{\lambda_P}\right)}{\lambda_D - \lambda_P}$$

Also $\left[d^2[D]/dt^2 \right]_{tmax} < 0$