Solution to problem no. 44

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xy^2 = 1 - x

y^2 = 1/x - 1 ----- (1)

We can make the following observations abt the curve

1. It is symmetrical about the x - axis

2. At y = 0 , 1/x - 1 = 0

x = 1

ie., the curve cuts the x - axis at the point (1, 0)

3. LHS is always + ve,

(1/x - 1) ≥ 0

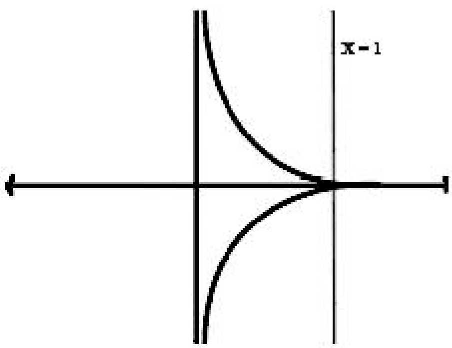
x ≤ 1 ie., Curve exists for x ≤ 1

4. As x--> 0 , y--> ∞

5. The curve does not exist for x < 0.

6. Therefore domain of f (x) is 0 < x ≤ 1

Curve can be sketched as shown :



y = [x / 2 – (x) / 3] . . . . (2)

The fractional part function is defined for x > 0 as

(x) = x – [x]

1.. At x = 1

(x) = 0

y = [1 / 2] = 0

2.. When 0 < x ≤ 1

(x) = x – [x]

(x) = x – 0 = x

y = [x / 2 – x / 3]

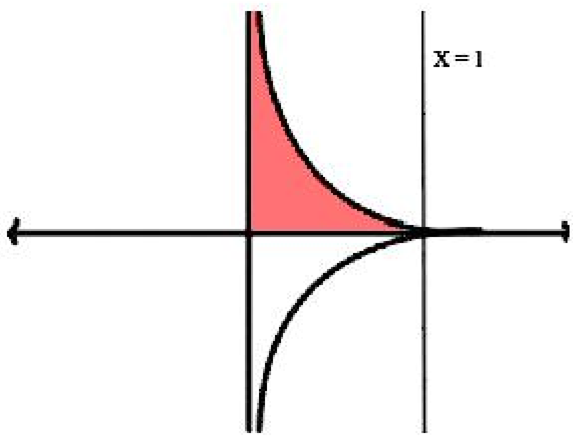
y = [x / 6]

0 < x ≤ 1,   0 < x / 6 ≤ 1 / 6

ie   y = [x / 6] = 0

Eqtn . (2) represents the x – axis in the interval 0 < x ≤ 1

The region whose are needs to be calculated is shown in red :



Area = Lim m → ∞ $\int_0^m$ f (y) dy

= Lim m → ∞ $\int_0^m$ [(y ^ 2 + 1) ^ –1] dy

= Lim m → ∞ $\tan^{-1}(y)$

= $\tan^{-1}(\infty)$

Area = π / 2 square units
