Solution to Problem No. 36:

In the above problem, the total number of capacitor plates are *n*, so the total number of capacitors in series are $(n-1)$.
For the first capacitor with plate areas A and A/2, the effective area for this capacitor will be taken to be A/2, that of A/2 and A/4, the effective capacitor area will be A/4 and so on.

All the capacitor plates are connected in series.

For the effective capacitance in series, we have the general formula:

$$1/C_{effective} = 1/C_1 + 1/C_2 + 1/C_3 + \ldots\ldots\ldots\ldots + 1/C_n$$

For each capacitor of plate area *A* and plate separation *d*, we have the general capacitance formula as $C = \varepsilon_0 A/d$ where ε_0 is called the permittivity constant whose value is $8.85 \times 10^{-12}\ C^2/N\text{-}m^2$.

(a)

Hence for the given system we have

$$1/C_{effective} = d/\varepsilon_0(A/2) + d/\varepsilon_0(A/4) + \ldots\ldots\ldots + d/\varepsilon_0(A/2^{n-1})$$

$$1/C_{effective} = \{d/\varepsilon_0 A\}\{2 + 2^2 + 2^3 + \ldots\ldots\ldots\ldots + 2^{n-1}\}$$

The above expression contains a geometric progression with $(n-1)$ terms. Using the formula of geometric equation to it, we get

$$1/C_{effective} = \{d/\varepsilon_0 A\}\{2\}\{2^{n-1} - 1\}$$

Hence the effective capacitance is

$$C_{effective} = \{\varepsilon_0 A/d\}/\{2^n - 2\}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{ANSWER}$$

(b)
For the second part, we first calculate the effective capacitance of the combination.

$$1/C_{effective} = d/k\varepsilon_0(A/2) + d/\varepsilon_0(A/4) + \ldots\ldots\ldots + d/\varepsilon_0(A/2^{n-1})$$

$$1/C_{effective} = 2d/k\varepsilon_0 A + 4\,d/\varepsilon_0 A + \ldots\ldots\ldots + 2^{n-1} d/\varepsilon_0 A$$

Taking the factor *2* common from the second expression, we get

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$$1 / C_{effective} = 2d / k\varepsilon_0 A + \{2d / \varepsilon_0 A\}\{2 + 2^2 + 2^3 + \ldots\ldots\ldots + 2^{n-2})$$

The second expression again consists of a GP but with $(n – 2)$ terms. Hence applying the formula for sum of a geometric progression and taking the factor $\{2d / \varepsilon_0 A\}$ common, we get

$$1 / C_{effective} = \{2d / \varepsilon_0 A\}\{(1/k) + 2(2^{n-2} – 1)\}$$

$$1 / C_{effective} = \{2d / k\varepsilon_0 A\}\{1 + k(2^{n-1} – 2)\}$$

Hence,

$$C_{effective} = \{k\varepsilon_0 A/2d\}\{\ 1/[\ 1 + k(2^{n-1} – 2)]\ \}$$

Also, if Q is the charge stored in the system when emf E is applied across the assembly, then the charge is given by

$Q = EC_{effective}$

Hence the charge stored in the capacitor combination in the second case is

$Q = \{k\varepsilon_0 AE/2d\}\{\ 1/[\ 1 + k(2^{n-1} – 2)]\ \}$ ……………………. ANSWER

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