

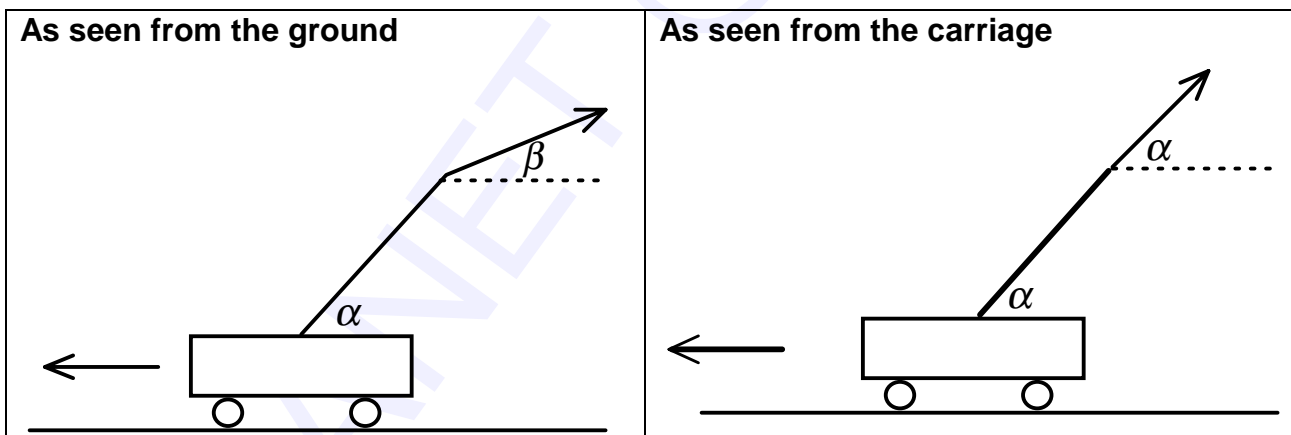
QUESTION 31

A gun is mounted on a gun carriage movable on a smooth horizontal plane. The gun is elevated at an angle α to the horizontal. A shot is fired which leaves the gun in a direction inclined at an angle β to the horizontal. If the mass of gun+carriage is n times that of the shot, show that

$$\tan \beta = \left(1 + \frac{1}{n}\right) \tan \alpha.$$

SOLUTION TO QUESTION 31

When the shot is fired from the gun, the shot should move at an angle α as seen from the carriage. However, since the carriage itself moves to the left in the horizontal direction due to recoil, the shot appears to move at an angle β as seen from the ground.



Let $v_{s/e}$ = velocity of shot wrt earth.

$v_{c/e}$ = (velocity of gun + carriage) wrt earth after the shot along the horizontal to the left.

Since the ground is smooth frictional forces can be neglected. Also no other external forces are acting on the system (shot+gun+carriage) with earth as reference, along horizontal. Conserving linear momentum of the system wrt earth

(mass of gun+carriage) \times v_c/e + (mass of shot) \times $v_s/e \cos\beta = 0$; (Initially the system is at rest)

$$\Rightarrow v_c/e = -(v_s/e \cos\beta)/n \quad [\text{since mass of gun+carriage} = n(\text{mass of shot})]$$

Now, the gun is inclined at angle α wrt ground.

$$\begin{aligned} \text{wrt the carriage, velocity of the shot along horizontal} &= v_s/e \cos\beta - (-v_c/e) \\ &= v_s/e \cos\beta (1 + 1/n) \quad \text{---(1)} \end{aligned}$$

$$\text{wrt the carriage, velocity of the shot along vertical} = v_s/e \sin\beta \quad \text{---(2)}$$

$\tan\alpha = \text{velocity along vertical} / \text{velocity along horizontal}$

$$\tan\alpha = (2) / (1)$$

$$\Rightarrow \tan\beta = (1 + 1/n) \tan\alpha$$

\Rightarrow which is the required answer.