SOLUTION TO QUESTION 29

Let us consider the following expression as:

X $= a^2 (b+c-a) + b^2 (c+a-b) + c^2(a+b-c) - 3abc$

$= a^2 (b+c-a) - abc + b^2 (c+a-b) - abc + c^2(a+b-c) - abc$

$= a[ab+ac-a^2-bc]+b[bc+ab-b^2-ac]+c[ac+cb-c^2-ab]$

$= a[a(b-a)-c(b-a)]+b[c(b-a)-b(b-a)] + c[a(c-b)-c(c-b)]$

$= a(b-a)(a-c)+b(b-a)(c-b)+c(c-b)(a-c)$…………(A)

$= (b-a)[a^2-ac+bc-b^2] + c(c-b)(a-c)$

$= (b-a) [(a-b)(a+b)- c(a-b)]+c(a-c)(c-b)$

$= (b-a)[(a-b)[a+b-c]+ c(a-c)(c-b)$

$= -(a-b)^2(a+b-c)+ c(a-c)(c-b)$

= (-ve value) + c(a-c)(c-b)---→1

Since, a,b,c are the sides of a triangle
(sum of 2 sides of a triangle is greater than the third side.)
$a+b>c$
Also,
X $= (c-b)[b(b-a)+c(a-c)]+a(b-a)(a-c)$

$= (c-b)[b^2-ab+ac-c^2]+a(b-a)(a-c)$

$= (c-b)[(b-c)(b+c) – a(b-c)] + a (b-a)(a-c)$

$= -(b-c)^2[b+c-a]+a(b-a)(a-c)$

Since a,b,c are the sides of a triangle
(sum of 2 sides of a triangle is greater than the third side.)

$b+c>a$
X= (-ve value) + a(b-a)(a-c) ---→2

X $= (a-c)[a(b-a)+c(c-b)]+b(b-a)(c-b)$

$= a-c[ab-a^2+c^2-bc] + b(a-c)+(c+a)(c-a)$

$= (a-c)[(c-a)(c+a)+b(a-c)]$

$= -(c-a)^2[c+a-b]+b(b-a)(c-b)$ ---------→3

Since, a,b,c are the sides of a triangle

(sum of 2 sides of a triangle is greater than the third side.)

$c+a>b$

Adding the equations 1,2,3.

$3X = (\text{- ve value})+b(b-a)(c-b)+a(b-a)(a-c)+c(a-c)(c-b)$

$2X = (\text{- ve value})$ (from A)

$X = \text{- ve value}$

Then, in the case of an equilateral triangle,

$a=b=c$

$X = 0$

$X \leq 0$

Hence the result.