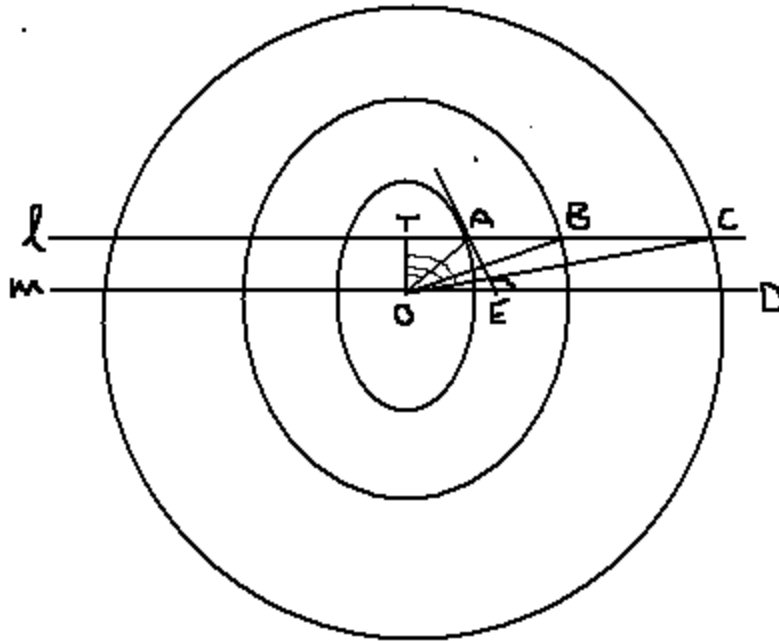


According to the question,
 The area of the triangle formed by the tangents to the circle at points A, B and C must be proved equal to $\frac{1}{2}p \cdot AB \cdot BC \cdot CA$.



In the above figure, l is the line talked about. $OT = p$; where O is the centre of the concentric circles and m is a line $\parallel l$ & passing through O .

First of all let's talk about the co-ordinates of the point A.
 Let O be the origin.

The y-coordinate $y=p$; Let $\angle AOD = a_1, \angle BOD = a_2, \angle COD = a_3$;

The x-coordinate $x=TA = \tan(\frac{\pi}{2} - a_1) \cdot p = p \cdot \cot a_1$;

The slope of the tangent AE to the point A is $\tan(\frac{\pi}{2} + a_1) = -\cot(a_1)$;.....[External Angle Theorem]

Therefore the equation of the tangent at A can be obtained by the slope-point form of a line as

$$x \cot a_1 + y - p(1 + \cot^2 a_1) = 0 \dots\dots\dots(1)$$

|||ly, the equation of the tangent at B is

$$x \cot a_2 + y - p(1 + \cot^2 a_2) = 0; \dots\dots\dots(2)$$

Now solving the above two equations by the rule of cross multiplication, we get

$$x = p(\cot a_2 + \cot a_1), y = p(1 + \cot a_1 \cot a_2);$$

This is the same as $x = TA + TB; (p \cot a_2 = TB \text{ and } p \cot a_1 = TA)$

$$\& y = (p^2 + OA \cdot OB) / p;$$

$$\text{Now, } x_1 = TA + TB, x_2 = TB + TC, x_3 = TC + TA;$$

$$\text{And } y_1 = (p^2 + TA \cdot TB) / p, y_2 = (p^2 + TB \cdot TC) / p,$$

$$y_3 = (p^2 + TC \cdot TA) / p;$$

where x_1, x_2, x_3 are the x-coordinates of the three meeting points of the tangents and y_1, y_2, y_3 are the y-coordinates of the points.

Now the area of the required triangle is, using determinants

$$\begin{vmatrix} 1 & x_1 & y_1 & 1 \\ 1 & x_2 & y_2 & 1 \\ 1 & x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2} \begin{vmatrix} TA + TB & (p^2 + TA \cdot TB) / p & 1 \\ TB + TC & (p^2 + TB \cdot TC) / p & 1 \\ TA + TC & (p^2 + TA \cdot TC) / p & 1 \end{vmatrix}$$

After taking $1/p$ common from the 2nd column and applying $R_1 \rightarrow R_1 - R_2$ and $R_2 \rightarrow R_2 - R_3$, take $(TC - TA)$ and $(TB - TA)$ common and then evaluating, we get

$$\text{Area} = \frac{1}{2} p \cdot (TC - TA) \cdot (TB - TA) \cdot (TC - TB) = \frac{1}{2} p \cdot AC \cdot AB \cdot BC ;$$

Hence the result.

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