

Solution to Q.24

Let us displace the block of mass 'm' downwards through a distance 'x'

Let us also assume net force 'F' acting upward on the block

Then for the block,

$$F = T$$

Where T is the increase in the tension in the string

As the string is same all over (assuming light), the increase in tension will also be same everywhere

For the pulley1 (numbering is done in accordance with k_1, k_2, k_3, k_4)

$$k_1 \cdot x_1 = 2T = 2F \quad \text{----eq1}$$

For the pulley2

$$K_2 \cdot x_2 = 2T = 2F \quad \text{----eq2}$$

For the pulley3

$$K_3 \cdot x_3 = 2T = 2F \quad \text{----eq3}$$

For the pulley4

$$K_4 \cdot x_4 = 2T = 2F \quad \text{----eq4}$$

And also when pulley 1 is shifted through a distance 'x₁' then the block will displace through 2.x₁

Similarly for other pulleys

$$\text{Then } x = 2(x_1 + x_2 + x_3 + x_4) \quad \text{-----eq5}$$

Putting the value of eq1-4 in eq5

$$x = 4F \left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \frac{1}{k_4} \right)$$

$$\Rightarrow F = \frac{x}{4 \left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \frac{1}{k_4} \right)} = kx$$

As force $\propto x$ and is in opposite direction (x is downwards, whereas force is upwards) so the block will perform S.H.M.

$$\text{So } T = 2\pi \sqrt{\frac{m}{k}}$$

$$T = 2\pi\sqrt{4m\left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \frac{1}{k_4}\right)}$$

$$\Rightarrow T = 4\pi\sqrt{m\left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \frac{1}{k_4}\right)}$$

This is the time period of the S.H.M.